AP Calculus Summer Assignment

Welcome to AP Calculus. This summer assignment will help you review some Algebraic and Trigonometric topics that you will need for some problems in Calculus. This summer assignment is designed to help you review/relearn topics that we feel you need refreshed.

It is important that you understand the algebraic, exponential, trigonometric topics that are in the packet. Take your time doing the problems. The completed packet is due the first day of school. You will be tested on this material the first week of school.

You will need your function notebook. Please make sure it is up to date. If not, please copy the pages you do not have from the pages posted. You will also have a quiz on the parent functions the first week of school. You should be able to recognize the basic graphs as well as the trig, exponential and log graphs.

We are looking forward to working with you in September.

I. Basic Algebraic Rules

1. Are the following statements true? If not, change them to make them true.

a) $\dfrac{2k}{2k+4} = \dfrac{k}{k+4}$

b) $\dfrac{1}{p+q} = \dfrac{1}{p} + \dfrac{1}{q}$

c) $\dfrac{x+y}{2} = \dfrac{x}{2} + \dfrac{y}{2}$

d) $3\left(\dfrac{a}{b}\right) = \dfrac{3a}{3b}$

e) $3\left(\dfrac{a}{b}\right) = \dfrac{3a}{b}$

f) $3\left(\dfrac{a+b}{c}\right) = \dfrac{3a+b}{c}$

g) $\dfrac{x^2+1}{x-1} = x+1$

II. Complex Fractions & Rational Expressions

2. Simplify.

a) $\dfrac{\dfrac{x}{2}}{\dfrac{x}{4}}$

b) $h \div \dfrac{x+h}{h}$

c) $\dfrac{\sqrt{x-2} + \dfrac{5}{\sqrt{x-2}}}{x-2}$

3. Write as a single fraction with the denominator in <u>factored form</u>.

a) $\dfrac{7x^2+5x}{x^2+1} - \dfrac{5x}{x^2-6}$

b) $20\left(\dfrac{2}{x+1} - \dfrac{3}{x}\right)$

c) $x(1-2x)^{-3/2} + (1-2x)^{-1/2}$

d) $(3x-2)^{1/2} + x(3x-2)^{-1/2}$

e) $\dfrac{\dfrac{2}{x}-3}{1-\dfrac{1}{x-1}}$

4. Evaluate without a calculator:

   a. $\left(64^{5/6}\right)$ b) $\left(16^{3/2}\right)$ c) $\left(27^{5/3}\right)$ d) $\left(32^{7/5}\right)$

III. Negative and Fractional Exponents

5. Simplify using only positive exponents. Do not rationalize the denominators.

   α) $\dfrac{\sqrt{4x-16}}{\sqrt[4]{(x-4)^3}}$ b) $\left(\dfrac{1}{x^{-2}}+\dfrac{4}{x^{-1}y^{-1}}+\dfrac{1}{y^{-2}}\right)^{-1/2}$ c) $\left(\dfrac{x^{-2}}{y^{-1}}-x\right)^{-3}$

IV. Solving Equations and Factoring

6. Solve for y' in _simplest form_.

   a. $xy' + y = 1 + y'$ b. $3y^2y' + 2yy' = 5y' + 2x$ c. $3x^2yy' + 2xy^2 = 2yy'$

7. Solve the quadratic equation. Use any means from algebra: factoring, quadratic formula, graphing. Be sure answers are simplified. If you use graphing, state what you did on the calculator.

   a. $4x^2 - 21x - 18 = 0$ b. $2x^2 - 3x + 3 = 0$ c. $x^4 - 9x^2 + 8 = 0$

8. Factor completely (There should be negative exponents in some of the problems.)

a. $3x^3 + 192$
b. $9x^2 - 3x - 2$
c. $2\sqrt{x} - 6x^{\frac{3}{2}}$

d. sin x + tan x
e. $e^{-x} - xe^{-x} + 2x^2 e^{-x}$

V. Equations of lines

9. Find the equation of the line that passes through the point (2, 4) and is parallel to the line 2x + 3y – 8 = 0.

10. Find the equation of the line that is perpendicular to the line 2x + 3y – 8 = 0 at the point (1, 2).

11. The line with slope 5 that passes through the point (-1, 3) intersects the x-axis at a point. What are the coordinates of this point?

12. What are the coordinates of the point at which the line passing through the points (1, -3) and (-2, 4) intersects the y-axis?

13. Graph the equation $y = x^3 - x$ and answer the following questions.

    a. Is the point (3, 2) on the graph?

    b. Is the point (2, 6) on the graph?

    c. Is the function odd, even or neither?

    d. Find the x and y – intercept(s).

VI. Asymptotes and Intercepts

14. Find all intercepts and asymptotes. (Just the VA and HA's.)

    a. $y^2 = x^2 - 4x$

    b. $y = \dfrac{x^2 + 3x}{(3x+1)^2}$

    c. $y = \dfrac{x^2 - 4}{x^2 - x - 12}$

    d. $y = \dfrac{3x - 1}{2x^2 + x - 6}$

VII. Domain

15. Use interval notation to identify the domain for each of the following functions.

    a. $h(x) = \dfrac{1}{4x^2 - 21x - 18}$

    b. $k(x) = \sqrt{x^2 - 5x - 14}$

c. $\dfrac{\sqrt[3]{x-6}}{\sqrt{x^2-x-30}}$

d. $d(x) = \ln(2x-12)$

VIII. Functions

17. Find f(1) – f(5) given f(x) = |x – 3| - 5.

18. Find f(x + 2) – f(2) given f(x) = $x^2 - 3x + 4$.

19. Find f(x + h) for f(x) = $x^2 - 2x - 3$.

20. Find $\dfrac{f(x+h)-f(x)}{h}$ if f(x) = $\sqrt{8x^2} + 1$.

21. Given f(x) = x – 3 and g(x) = x, complete the following.
   a. f(g(x)) =   b. g(f(x)) =   c. f(f(x)) =

22. Given $f(x) = \dfrac{1}{x-5}$ and g(x) = $x^2 - 5$, complete the following.

   a. f(g(7))   =   b. g(f(v)) =   c. g(g(x)) =

IX. Binomial Theorem
23. Use the Binomial Theorem to expand and simplify the following expressions.

   a. $(x + 5)^5$ b. $(3a - 4b)^4$ c. $(2w + 3)^6$

X. Factor Theorem

24. Use the p over q method and synthetic division to factor the polynomial P(x). Then solve P(x)=0.

   a. $P(x) = x^3 + 5x^2 - 2x - 24$

   b. $P(x) = x^4 + 5x^3 + 6x^2 - 4x - 8$

XI Logarithms

25. Condense the expression: $2\ln(x-3) + \ln(x+2) - 6\ln x$

26. Expand the expression: $\ln\dfrac{x^2\sqrt{y-2}}{3w^3 z}$

27. Express y in terms of x.

   a. ln y = x + 2

   b. ln y = 2lnx + ln10

   c. ln y = 4lnx + 3

   $x = \ln\dfrac{e^{x^2}}{4y}$ $x = \ln\dfrac{e^{x^2}}{4y}$ $x = \ln\dfrac{e^{x^2}}{4y}$ d.

   d. $x = \ln\dfrac{e^{x^2}}{4y}$

28. Solve for x.

   a. $\ln e^3 = x$ b. $\ln e^x = 4$ c. lnx + lnx = 0   d. $e^{\ln 5} = x$

   e. ln1 - lne = x   f. ln6 + lnx - ln2 = 3   g. ln(x + 5) = ln(x - 1) - ln(x +1)

XII. Trigonometry

29. Evaluate (without a calculator!!). NO decimals.

   a. $\cos 0$

   b. $\sin 0$

   c. $\tan \dfrac{\pi}{2}$

   d. $\cos \dfrac{\pi}{4}$

   e. $\sin \dfrac{\pi}{2}$

   f. $\sin \pi$

   g. $\sin^{-1} \dfrac{\sqrt{3}}{2}$

   h. $\tan^{-1} 1$

   i. $\tan^{-1} \dfrac{\sqrt{3}}{3}$

   j. $\sec^{-1} \sqrt{2}$

   k. If $\cos\theta = \dfrac{5}{13}$ and θ is in Quadrant II, Find the all the remaining trig functions.

30. Which of the following expressions are identical?

   a. $\cos^2 x$

   b. $(\cos x)^2$

   c. $\cos x^2$

31. Which of the following expressions are identical?

   a. $(\sin x)^{-1}$

   b. BCDEFG'

   c. $\sin x^{-1}$

   d. $\dfrac{1}{\sin x}$

32. Solve the following for the indicated variable on the interval $[0, 2\pi)$.

   a. $3\cos x - 1 = 2$

   b. $2\sin(2\theta) - \sqrt{3} = 0$

   c. $\tan^2 x - 1 = 0$

   d. $2\sin^2 x + \sin x = 1$

33. Complete the following trig identities

   a. $\sin^2 x + \cos^2 x =$

   b. $\tan^2 x + 1 =$

   c. $\dfrac{1 - (\sin\alpha + \cos\alpha)^2}{2\sin\alpha}$

XIIIWord Problems

34. Find the surface area of a box of height h whose base dimensions are p and q and satisfies the following conditions:
    a. The box is closed.

    b. The box has an open top.

    c. The box has an open top and a square base with side length p.

35. A seven foot ladder, leaning against a wall, touches the wall x feet above the ground. Write an expression in terms of x for the distance from the foot of the ladder to the base of the wall.

36. A piece of wire 5 inches long is to be cut into two pieces. One piece is x inches long and is to be bent into the shape of a square. The other piece is to be bent into the shape of a circle. Find an expression for the total area made up by the square and the circle as a function of x.

37. A police car receives a radio call to catch a vehicle which is speeding down the highway at 80 mph. The police car, which is 12 miles away, drives after it at 108 mph. How long will it take for the police car to catch up?

38. The base of a triangle is 6 cm more than the height. If the area of the triangle is 140 square cm, what is the length of the base?

39. Two trains, the Express and the Commuter, leave the same station at the same time. The Express, which heads north, travels 10 km per hour faster than the Commuter, which goes east. If the trains are 100 km apart after 2 hours, find the speed of each train.

40. The depth, d, of a buoyant object t seconds after plunging into water can be found using the equation $d = -6t^2 + rt$, where r is the velocity at which the object strikes the water. If the object strikes the water at a velocity of 240 feet per second, find the maximum depth the rocket will reach and at what time. When will the rocket surface again?

41. Find the average rate of change (i.e. slope) for the following functions on the indicated intervals.

a. $f(x) = x^3 - 2x; [0, 4]$

b. $f(x) = 3\sqrt{x}; [4, 25]$

c. A car travels 420 miles over a period of 210 minutes. Find the average velocity of the car in miles per hour over this time period.

d. On January 1st 2003, the value of a stock was $135 per share. By December 1st 2003, the value of the stock had fallen to $38 per share. What is the average rate of change in the value of the stock in dollars per month?

e. In 1984, the Fizzy Cola Company sold 23 million gallons of soda. By 2003, the company was selling 127 million gallons of soda. What is the average rate of change in the number of gallons of soda sold per year?

f. During a recent trip to the store, a car's velocity went from 0 to 60 mph in 20 seconds. What is the average acceleration of the car in miles per hour per hour?